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Geometry

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COURSE OVERVIEW

Description

All levels of Geometry are a course in plane geometry, with some aspects of solid geometry included. Moving towards formal mathematical arguments, the standards in high school geometry are meant to formalize and extend what was learned in middle school geometry. The nature of deductive proof is emphasized; logical reasoning is applied. Transformations are presented early in the year to build conceptual understanding of geometric concepts throughout the year. Students will also study congruence, similarity, proportional reasoning, right triangle trigonometry, circles, surface area and volume. Essentials, Academic, and Honors utilize the same NJSLS content standards. Technological tools assist in illustrating the connections between geometry and other areas of mathematics, and demonstrate the power of mathematics. Algebra practice will also be embedded throughout the course so that students are prepared appropriately for Algebra 2 and Precalculus.

Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics. The eight Standards for Mathematical Practice are outlined below:

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

Scope and Sequence			
11	Торіс	Length	
Unit		HS	MS
Unit 1	Tools of Geometry	7 days	14 days
Unit 2	Reasoning and Proof	5 days	10 days
Unit 3	Parallel and Perpendicular Lines	8 days	16 days
Unit 4	Transformations	8 days	16 days
Unit 5	Congruence	9 days	18 days
Unit 6	Similarity	9 days	18 days
Unit 7	Quadrilaterals	8 days	16 days
Unit 8	Trigonometry	11 days	22 days
Unit 9	Circles	8 days	16 days
Unit 10	Surface Area and Volume	7 days	14 days
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Resources Core Text: Geometry (2022) Ron Larson, Laurie Boswell Big Ideas Learning, LLC (bigideasmath.com) Suggested Resources: IXL, Delta Math, Quizizz, Gimkit, EdPuzzle, GeoGebra, desmos.com; KutaInfinite Geometry; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulator

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UNIT 1: Tools of Geometry

Summary and Rationale

This introductory unit brings the concepts and skills learned through middle school geometry into a more formal setting by introducing constructions and the vocabulary of geometry. Students will begin the process of developing the definitions, postulates and theorems used to prove geometric statements and will learn about different types of reasoning that can be applied. Constructions using technology or completed by hand will allow students to understand the logic behind many of the concepts learned previously and apply it to find areas and perimeters of polygons in the coordinate plane.

Recommended Pacing

7 days

State Standards

Standard G- CO Congruence

CPI #	Cumulative Progress Indicator (CPI)
1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
Standa	rd G-GPE Expressing Geometric Properties with Equations
CPI #	Cumulative Progress Indicator (CPI)
7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
Standa	rd G-MG
CPI #	Cumulative Progress Indicator (CPI)
1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
	Instructional Focus
Unit Er	nduring Understandings
 Log The Pos 	gical arguments can be made based on known or deduced information. e basics of geometry is used to describe relationships between points, lines and rays in planes. stulates and theorems are used to show the relationships of bisectors and distances.
Unit Es	sential Questions
• Ho	w can you use reasoning to solve problems?
• In a	a diagram, what can be assumed and what needs to be labeled?
Object	ives
Studen	its will know:
• Ho	w to describe points, lines and planes.
 Ho 	w to construct line segments and their bisectors, angles and their bisectors.

- How to measure line segments and angles.
- **Vocabulary:** undefined terms, point, line, plane, collinear points, coplanar points, defined terms, line segment, endpoints, ray, opposite rays, intersection, postulate, coordinate, distance between two points, construction,

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Mathematics: Curriculum

congruent segments, between, midpoint, segment bisector, angle, vertex, sides of an angle, interior and exterior of an angle, measure of an angle, acute angle, right angle, obtuse angle, straight angle, congruent angles angle bisector, adjacent angles, complementary angles, supplementary angles, linear pair and vertical angles.

Students will be able to:

- Name segments, intersecting lines, intersecting planes and lines that are not coplanar from a given diagram.
- Sketch intersections of lines and planes.
- Describe what a postulate is and provide an example of a postulate.
- Students will be able to use the segment addition and angle addition postulates along with the distance formula to find the length of segments.
- Construct a segment bisector and angle bisector.
- Find the midpoint of a segment using the midpoint formula.
- Classify and describe polygons.
- Find the perimeter and area of polygons in the coordinate plane using the distance formula.
- Measure and classify angles.
- Construct congruent angles.
- Identify complementary and supplementary angles.
- Identify linear pairs and vertical angles.
- Find angle measures in pairs of angles.

Resources

UNIT 2: Reasoning and Proof

Summary and Rationale

This unit brings the concepts and skills learned through middle school geometry into a more formal setting by introducing formal proof. Students will discover how proof and structured argument form the basis for future analysis. The unit introduces inductive and deductive reasoning as well as conditional statements written in if-then form. Students will determine if conditional statements are true or false. Several postulates will be introduced so students can begin writing proofs about lines, angles, triangles, and parallelograms. Algebraic proof will be introduced to make connections back to previous knowledge of solving equations from Math 8 and Algebra 1.

Recommended Pacing

5 days		
	State Standards	
Standa	rd G-CO Congruence	
CPI #	Cumulative Progress Indicator (CPI)	
1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	
	Instructional Focus	
Unit Er	nduring Understandings	
 The loce Ge Procession Log Unit Essent Ho Ho Ho In a Ho 	e process of proving includes developing conjectures, considering a general case, exploring with examples, king for structural similarities across cases, and searching for counterexamples. ometry uses a wide variety of kinds of proofs. bofs of theorems are a form of deductive reasoning gical arguments can be made based on known or deduced information sential Questions w can we use properties of plane figures to make logical arguments about geometric relationships? hen is a conditional statement true or false? w can you use reasoning to solve problems? a diagram, what can be assumed and what needs to be labeled? w can you prove a mathematical statement?	
Object	ives	
Studen Ho Ho Vo bic sta	Its will know: w to approach formal proofs using a variety of methods w to model and solve problems related to geometric proofs cabulary: point, line, plane, terms related to logical reasoning (conjecture, theorem, conditional statement, onditional, and counterexample), terms related to proof (two column, flow, paragraph, conjecture, tement, reason)	
Studen	Students will be able to:	
• Exp	plain the purpose of using conditional and biconditional statements in the context of formal proof (2.1)	

- Write conditional and biconditional statements and determine their truth validity. (2.1)
- Identify and sketch diagrams of point, line, and plane postulates (2.3)

- Complete formal proofs (two-column, flow, or paragraph) of algebraic problems using properties and postulates to justify statements (2.4)
- Complete formal proofs (two-column, flow, or paragraph) using vertical angles, angle and segment bisectors, linear pairs, complementary and supplementary angles, midpoints, segment and angle addition postulates (2.5)

Resources

UNIT 3: Parallel and Perpendicular Lines

Summary and Rationale

This unit has students using prior knowledge of slope and equations of lines to prove two lines are either parallel or perpendicular or neither. Students will learn about the angle relationships that occur when two lines are cut by a transversal. They will also be able to prove that when the lines are parallel the angle relationships would either be congruent or supplementary.

Recommended Pacing

8 days		
	State Standards	
Standa	rd G-CO Congruence	
CPI #	Cumulative Progress Indicator (CPI)	
1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	
12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). <i>Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i>	
Standa	rd G-GPE Expressing Geometric Properties with Equations	
CPI #	Cumulative Progress Indicator (CPI)	
5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
	Instructional Focus	
Unit Enduring Understandings		
• Ge	ometry uses a wide variety of kinds of proofs.	
• Pro	oofs of theorems are a form of deductive reasoning	
• Log	gical arguments can be made based on known or deduced information	
Unit Es	sential Questions	
• Ho	w can we use counterexamples to identify flaws in proofs and definitions?	
• Wł	tools can be used to perform constructions?	
Obiect	ives	
Studen	its will know:	
• Ho	w to understand lines, planes, and pairs of angles.	
• Ho	w to prove and use theorems about parallel lines.	
• Ho	w to prove and use theorems about identifying parallel lines.(Converse Theorems)	
 Ho 	w to prove and use theorems about perpendicular lines.	

Mathematics: Curriculum

- How to partition a directed line segment and understand slopes of parallel and perpendicular lines.
- Vocabulary: parallel lines, skew lines, parallel planes, transversal, corresponding angles, alternate interior angles, alternate exterior angles, consecutive interior angles, distance from a point to a line, perpendicular bisector, directed line segment, slopes of parallel lines, and slopes of perpendicular lines.

Students will be able to:

- Identify lines and planes.(3.1)
- Identify parallel and perpendicular lines.(3.1)
- Identify pairs of angles formed by transversals.(3.1)
- Use properties of parallel lines to find angle measures.(3.2)
- Prove theorems about parallel lines.(3.2)
- Use theorems to identify parallel lines.(3.3)
- Construct parallel lines.(3.3)
- Prove theorems about identifying parallel lines. (Converse Theorems) (3.3)
- Find the distance from a point to a line.(3.4)
- Construct perpendicular lines and perpendicular bisectors.(3.4)
- Prove theorems about perpendicular lines.(3.4)
- Partition directed line segments using slope.(3.5)
- Use slopes to identify parallel and perpendicular lines.(3.5)
- Write equations of parallel and perpendicular lines.(3.5)
- Find the distance from a point to a line.(3.5)

Resources

UNIT 4: Transformations

Summary and Rationale

In this unit students begin to define congruence and similarity in terms of transformations. Students will learn that rigid motions preserve distance and angle measure, whereas nonrigid transformations may change the shape or size of a figure. Students will also explore transformations on the coordinate plane to better understand distance and angle relationships.

Recommended Pacing

8 days	
	State Standards
Standa	ard G-CO Congruence
CPI #	Cumulative Progress Indicator (CPI)
2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments
5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
Standa	ard G-SRT Similarity, Right Triangles, and Trigonometry
CPI #	Cumulative Progress Indicator (CPI)
1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
	Instructional Focus
Unit E	nduring Understandings
 Co Co Rig 	ngruence can be verified using transformations ngruence is a special case of similarity. gid transformations preserve distance and angles
Unit E	ssential Questions
• Ho	w do rigid transformations preserve distance and angles? Are there other transformations that preserve one aracteristic or neither characteristic?

Mathematics: Curriculum

- How do coordinates of geometric figures change when rigid transformations are applied to them?
- How does the concept of rigid motion connect to the concept of congruence?
- How can transformations be used to explain similarity?
- What is the relationship between transformations that produce congruent figures and transformations that produce similar figures?

Objectives

Students will know:

- How to perform, notate, and analyze a variety of rigid transformations
- How to use the coordinate plane to examine different types of rigid transformations
- How to model and solve problems related to isometry and transformations
- How to perform, notate, and analyze dilations
- Vocabulary: Image, preimage, rigid transformation, translation, reflection, rotation, rigid motion, isometry, symmetry, dilation, similarity, scale factor

Students will be able to:

- Perform translations (using vectors), reflections, and rotations (4.1, 4.2, 4.3)
- Perform transformations both on and off the coordinate plane
- Use coordinate notation and function notation to map points from preimage to image and vice versa.
- Use slope and midpoint formulas to find equations of lines of reflection
- Identify lines of symmetry and angles of rotational symmetry for figures
- Perform sequences of transformations to map congruent figures onto one another
- Identify a sequence of coordinates involved in a sequence of transformations
- Define congruence through rigid transformations on the coordinate plane and validate using distance formula and midpoint formula
- Discuss perpendicular bisectors as related to reflections [all points on a perpendicular bisector equidistant from the endpoints of the segment (Perpendicular Bisector Theorem)] (4.4)
- Perform dilations completed from the origin and other points on coordinate plane (4.5)
- Find the scale factor and center of dilation given a dilation on a coordinate plane (4.5)
- Discuss similarity through the use of dilations (4.6)

Resources

Unit 5: Congruence

Summary and Rationale

In this unit, students delve more deeply into the concept of congruence, specifically examining triangle congruence in its many forms. Students will prove theorems about triangles and use the definition of congruence in terms of rigid motions to show that two triangles are congruent. Students will also explore other polygons through congruence. Proof and congruence will be explored on the coordinate plane so that students gain a formal understanding of how these relationships can be represented algebraically. Modeling will be used throughout the unit to show students applications of congruence in its many forms. Students will also explore the relationships within triangles including perpendicular bisectors, angle bisectors, medians, altitudes, and midsegments.

Recommended Pacing

9 days		
State Standards		
Standa	rd G-CO Congruence	
CPI #	Cumulative Progress Indicator (CPI)	
7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent	
8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	
Standa	rd G-SRT Similarity, Right Triangles, and Trigonometry	
CPI #	Cumulative Progress Indicator (CPI)	
5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
Standa	rd G-GPE Expressing Geometric Properties with Equations	
CPI #	Cumulative Progress Indicator (CPI)	
4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).	
	Instructional Focus	
Unit Er	nduring Understandings	
 The loc Co Co 	e process of proving includes developing conjectures, considering a general case, exploring with examples, oking for structural similarities across cases, and searching for counterexamples. ngruence can be verified using transformations ngruence is a special case of similarity.	
• Th	ere are many ways to approach a problem.	
Unit Es	ssential Questions	
 Wł Cal Ho 	nat are the properties of congruent triangles? What are the properties of congruent figures in general? n there be many ways to solve the same problem? w formal does a proof need to be for it to be considered complete?	
Object	ives	

Mathematics: Curriculum

Students will know:

- How to prove and analyze whether triangles are congruent
- Use triangle proof to explore and determine characteristics of quadrilaterals
- Explore triangle congruence and quadrilaterals on the coordinate plane in order to examine congruence
- How to model and solve problems related to congruence
- Vocabulary: isosceles, scalene, equilateral, acute, right, obtuse, median, altitude, corresponding, congruence theorem

Students will be able to:

- Explore the triangle congruence theorems (AAS, ASA, SSS, SAS, HL)
- Prove triangles congruent using two-column, paragraph, and coordinate proofs
- Use rigid motions to map congruent figures onto one another
- Use Corresponding Parts of Congruent Triangles are Congruent in proofs
- Justify angle bisector and perpendicular bisector constructions using triangle congruence
- Use the Triangle Sum Theorem (using auxiliary lines), Exterior Angle Theorem, Isosceles Triangle Theorem (Base Angle Theorem), Equilateral Triangle Theorem in proofs and to solve algebraic problems
- Use distance, midpoint, and/or slopes to prove triangles congruent in the coordinate plane (coordinate proofs)

Resources

Core Text: Geometry (2022) Ron Larson, Laurie Boswell Big Ideas Learning, LLC (bigideasmath.com)

Suggested Resources: IXL, Delta Math, Quizizz, Gimkit, EdPuzzle, GeoGebra, desmos.com; KutaInfinite Geometry; deltamath.com; <u>https://parcc.pearson.com/practice-tests/math</u>; collegeboard.org; graphing calculators and emulator

Unit 6: Similarity

Summary and Rationale

In this unit, students examine similarity and formalize the understanding of scale factor that was developed in middle school. In previous units students learn that two geometric figures are similar if and only if there is a similarity transformation that maps one of the figures to the other. Students will now explore other ways to show that two triangles are similar. Formal arguments and proofs will be made to justify that two triangles are similar by AA, SSS, or SAS. Students will apply proportional relationships in future units when discussing trigonometric ratios.

Recommended Pacing

8-9 days		
State Standards		
Stand	dard G-SRT Similarity, Right Triangles, and Trigonometry	
CPI #	Cumulative Progress Indicator (CPI)	
2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	
3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	
4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity	
5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures	
Stand	dard G-GPE Expressing Geometric Properties with Equations	
CPI #	Cumulative Progress Indicator (CPI)	
5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
	Instructional Focus	
Unit	Enduring Understandings	
• T (• T t	 The process of proving includes developing conjectures, considering a general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples. There are special relationships between the sides and angles of right triangles. These relationships can be used to solve problems involving missing side lengths or angle measures. 	
Unit	Essential Questions	
• F • F	How can transformations be used to prove similarity? How are the transformations used to prove congruence different from those used to prove similarity? How can we use ratios, proportions, and similarity to solve real world problems?	
Obje	ctives	
Stude • + • + • + • +	ents will know: How to determine and prove triangles and other polygons are similar both on and off the coordinate plane How ratios and proportions can be used to find missing sides and angles of similar figures How to model real world problems using similar figures /ocabulary related to similarity: ratio, proportion, scale factor, etc.	

• vocabalary rel

Students will be able to:

- Use ratios to determine missing side lengths of similar figures
- Determine if two figures are similar and write accurate similarity statements
- Use ratios to determine area and perimeter of similar figures
- Prove triangles similar using AA, SSS, and SAS similarity theorems
- Prove figures similar on the coordinate plane using similarity transformations
- Use the Triangle Proportionality Theorem to find missing pieces of similar triangles *Use the converse of the Triangle Proportionality Theorem to prove parallel lines
- Use other proportionality theorems (Three Parallel Lines and Triangle Angle Bisector) to set up proportions and solve for missing side lengths
- Use ratios, proportions, and similarity to solve real world problems

Resources

Unit 7: Quadrilaterals

Summary and Rationale

In this unit, students build upon their knowledge of the properties of different types of quadrilaterals. These properties related to sides, angles, and diagonals of quadrilaterals such as parallelograms, rhombuses, rectangles, and squares will be explored through formal proof and will also be shown on the coordinate plane in order for students to develop an algebraic understanding of the topic. Models using quadrilaterals will be used throughout the unit.

Recommended Pacing

7-8 days **State Standards Standard G-CO Congruence** CPI # **Cumulative Progress Indicator (CPI)** 11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals **Standard G-GPE Expressing Geometric Properties with Equations** CPI # **Cumulative Progress Indicator (CPI)** 4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2). **Instructional Focus Unit Enduring Understandings** The process of proving includes developing conjectures, considering a general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples. • As a quadrilateral becomes more unique, it maintains the properties of all the quadrilaterals that precede it. **Unit Essential Questions** What properties do parallelograms, rhombuses, rectangles, and squares share and why? Which properties are unique to each? How can we use the properties of quadrilaterals to accurately identify and prove a figure is a certain type of quadrilateral? How can we use what we know about transformations and congruence to help prove properties of quadrilaterals? Objectives Students will know: • The properties of the sides, angles and diagonals of a parallelogram, rectangle, rhombus, and square How to identify and prove different types of quadrilaterals both on and off the coordinate plane How quadrilaterals and their properties can be used to solve real world problems Students will be able to: Find the interior and exterior angle measures for any regular polygon Use formal proof to show that a quadrilateral is a parallelogram, rhombus, rectangle, or square Use distance, midpoint and slope to prove different types of quadrilaterals on the coordinate plane

- Identify quadrilaterals by the properties of their sides, angles, and/or diagonals
- Use the Trapezoid Midsegment Theorem both on and off the coordinate plane to find missing lengths
- Use properties of quadrilaterals to solve real world problems

Resources

Unit 8: Trigonometry

Summary and Rationale

In this unit, students apply their knowledge of similarity to explore relationships in triangles, which relates how the ratio between sides of triangles is affected by the angles within the triangle. Students will learn how to calculate trigonometric functions and their inverses and will explore how to solve for measurements within triangles. Students will also explore the Pythagorean Theorem and how its converse extends to being able to classify triangles by the relationship of the triangle's side lengths. Modeling will be used throughout the unit so that students understand the importance of trigonometric relationships in a variety of applications.

Recommended Pacing

10 - 11 days

State Standards

Standard G-SRT Similarity, Right Triangles, and Trigonometry		
CPI #	Cumulative Progress Indicator (CPI)	
4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	
5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	
6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	
7	Explain and use the relationship between the sine and cosine of complementary angles.	
8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	
9	(+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	
10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.	
11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	
	Instructional Focus	
Unit Er	nduring Understandings	
 The process of proving includes developing conjecture, considering a general case, exploring with examples, looking for structural similarities across cases, and searching for counterexamples. There are special relationships between the angles and sides of triangles, particularly right triangles. These relationships can be used to solve problems involving missing side lengths or angle measures. 		
Unit Essential Questions		
 Wł Ho 	nat is the relationship between the angles of a triangle and the side lengths of the triangle? w can trigonometry be used in indirect measurement or estimation?	
Objectives		
Students will know:		
 Ho 	w to identify and use special segments in triangles	

- How to use and prove the Pythagorean Theorem and its Converse
- How the properties of similar triangles relate to trigonometric ratios

- How to model and solve problems related to right triangle trigonometry and trigonometry in other real-world scenarios
- **Vocabulary:** equidistant, concurrent, altitude of a triangle, midsegment of a triangle, indirect proof, pythagorean triple. Trigonometric ratio, angle of elevation, angle of depression

Students will be able to:

- Identify and use perpendicular and angle bisectors of triangles
- Use medians and altitudes of triangles to solve problems
- Use the circumcenter, incenter, centroid, and orthocenter of a triangle
- Find distances using the Triangle Midsegment Theorem
- Compare measures within triangles and between two triangles
- Use the Pythagorean Theorem and its converse to solve problems
- Prove the Pythagorean Theorem and its converse
- Find side lengths in special right triangles
- Explain how similar triangles are used to define trigonometric ratios
 - Demonstrate and apply the relationship between the sine and cosine of complementary angles
 - Discuss the range of trigonometric ratios (i.e. sine and cosine cannot have a value over one)
- Use trigonometric ratios to solve problems
 - Include angles of elevation and depression
- Calculate area of triangle

Resources

Unit 9: Circles

Summary and Rationale

In this unit, students will refine their understanding of the definition of a circle. Students will look at segment and angle relationships within circles. Students will develop an understanding of arc measure, arc length, and sector area leading to a first understanding of radian measure. Within this unit, students will also look at circles on the coordinate plane in order to derive a formula and to understand the relationship between center and radius. Students will apply knowledge of circles and the segments and angles involving circles within modeling problems throughout the unit.

Recommended Pacing

7 - 8 days		
	State Standards	
Standa	rd G-CO Congruence	
CPI #	Cumulative Progress Indicator (CPI)	
1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	
Standa	rd G-C Circles	
CPI #	Cumulative Progress Indicator (CPI)	
1	Prove that all circles are similar.	
2	Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship</i> between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	
3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	
4	(+) Construct a tangent line from a point outside a given circle to the circle.	
5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	
Standa	rd G-GPE Expressing Geometric Properties with Equations	
CPI #	Cumulative Progress Indicator (CPI)	
1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
2	Derive the equation of a parabola given a focus and directrix.	
4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$	
Standa	Standard G-GMD Geometric Measurement and Dimension	
CPI #	Cumulative Progress Indicator (CPI)	
1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments	
Standa	rd G-MG	

СРІ	#	Cumulative Progress Indicator (CPI)
1		Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2		Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3		Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★
Uni	it En	during Understandings
•	The	process of proving includes developing conjectures, considering a general case, exploring with examples,
	loo	king for structural similarities across cases, and searching for counterexamples.
•	Seg	ments that can be drawn in a circle have a variety of relationships that can be explored.
•	Ang	gles that can be drawn in a circle have a variety of algebraic relationships that can be explored.
Uni	it Es	sential Questions
•	Wh	at is a circle?
•	Нο	w can knowledge of similar triangles help us discover relationships about segments and angles in circles?
•	Wh	at relationships can be formed between the Pythagorean Theorem, the Distance Formula, and the equation
	ofa	circle?
Ob	jecti	ves
Stu	den	ts will know:
•	Ho	w to describe and define different aspects of a circle, including its angles, segments, arcs, circumference,
	and	area
•	Exp	lore circles on the coordinate plane using equations and transformations
•	Ho	w to model and solve problems related to circles
•	Voo	abulary: circle, radius, diameter, concentric circles, central angle, minor arc, major arc, semicircle,
	con	gruent circles, inscribed angle, inscribed polygon, circumscribed circle, circumscribed angle, circumerence,
	reg	ular polygon
Stu	den	ts will be able to:
•	Pro	ve that all circles are similar
•	Ide	ntify Circle relationships
		 Relationship between inscribed, central, and circumscribed angles
		 Identify lines and segments that intersect circles
		 Use properties of tangents to solve problems
•	Use	e similarity to derive characteristics of chords, secants, and tangents
		• Find angle and arc measures in circles
		• Use chords of circles to find lengths
		• Describe the relationship between a diameter and a chord perpendicular to the diameter
•	Арр	bly and use constructions of inscribed and circumscribed polygons
		 Prove properties of angles for a quadrilateral inscribed in a circle (opposite angles are supplementary) sincles to use delivered bits angle bits angles are supplementary.
•	Use	circles to model and solve real-life problems
•	Use	The definition of a circle to derive an equation
	E:~	Complete the square to find radius and center
	LIU	\circ Define radians
		Kesources

Unit 10: Surface Area and Volume

Summary and Rationale

In this unit, students expand their understanding of solid figures (cones, cylinders, spheres, prisms, and pyramids) that was developed in middle school by discussing nets, cross sections, and various representations on the coordinate plane. Students will also explore Cavalieri's Principle and compound solid figures to examine solids more closely. Modeling will be explored throughout the unit. Students will calculate surface areas and volumes using multiple approaches.

Recommended Pacing

6 - 7 days	
	State Standards
Standa	rd G-GMD Geometric Measurement and Dimension
CPI #	Cumulative Progress Indicator (CPI)
1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures
3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. $igstarrow$
4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
Standa	rd G-GMD Geometric Measurement and Dimension
CPI #	Cumulative Progress Indicator (CPI)
1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
Unit Er	nduring Understandings
 The loc The The loc 	e process of proving includes developing conjectures, considering a general case, exploring with examples, king for structural similarities across cases, and searching for counterexamples. ree-dimensional figures can be examined through the process of creating nets.
• Tw sur	o dimensional objects can be used to construct three-dimensional objects allowing us to derive formulas for face area and volume.
Unit Es	sential Questions
 Ho Ho rea Ho rep 	w can we extend our knowledge of two dimensional objects to understand three dimensional objects? w can real-world objects be modeled by three-dimensional figures and how can this be helpful in solving Il-world problems related to the object? w can the formulas for volume, area and circumference be explained using geometric tools or physical presentations?
Objectives	
Studen • Ho • Ho	ts will know: w to extend their understandings of two-dimensional shapes to three-dimensional shapes w to derive volume and surface area formulas of various solids

- How to model and solve problems related to three-dimensional figures
- Vocabulary: center of a regular polygon, radius of a regular polygon, apothem of a regular polygon, central angle on a regular polygon, population density, face, edge, vertex, cross section, volume, similar solids, lateral surface of a cone, chord of a sphere, density, solids of revolution, axis of revolution

Students will be able to:

- Relationships between two and three dimensional shapes
- Describe attributes of solids
 - Describe and draw cross sections
- Find surface areas and volumes of solids and composite solids
- Find missing dimensions of solids
- Solve real-life problems involving surface area and volume
- Sketch and describe solids of revolution

Resources