# PISCATAWAY TOWNSHIP SCHOOLS 

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## Algebra 1

| Content Area: | Mathematics |
| :--- | :--- |
| Grade Span: | 7-9 |
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## COURSE OVERVIEW

## Description

Algebra 1 builds on concepts mastered in middle school mathematics classes and offers opportunities for students to develop as active problem solvers, critical thinkers, and effective communicators. The course requires students to explain their thinking and analyze diverse problems, while also providing students with the chance to develop mathematical reasoning to work through everyday mathematical challenges. Each unit of study provides students occasion to develop deeper understanding of mathematics coupled with gaining procedural skill and fluency and modeling applications as outlined in the Common Core State Standards. Major topics include graphical analysis and modeling with a variety of functions such as linear, exponential, absolute value and quadratic, and also includes data analysis and scatter plots. Students will learn techniques to simplify polynomial and radical expressions and will solve many types of equations and explore applications of systems of equations.

Additional topics may include graph analysis of square root and cube root functions (time permitting).

Standards that are related to modeling problems are marked with a star ( $\star$ ).

Topics marked with a double asterisk $\left(^{* *}\right)$ are to be covered in Honors Algebra 1 only.

## Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice set forth in the Common Core State Standards Initiative. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content.

The eight Standards for Mathematical Practice are outlined below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scope and Sequence

| Unit | Topic | Pacing HS | Pacing MS |
| :---: | :---: | :---: | :---: |
| Unit 1 | Arithmetic vs. Geometric Sequences | 5 days | 5 days |
| Unit 2 | Linear Functions, Graphs, and Tables | 15 days | 25 days |
| Unit 3 | Exponential Functions, Graphs, and Tables | 10 days | 15 days |
| Unit 4 | Absolute Value and Quadratic Functions | 20 days | 25 days |
| Unit 5 | Polynomial and Radical Operations | 15 days | 20 days |


| Unit 6 | Relating the Roots of a Quadratic Function to its Graph | 25 days | 35 days |
| :---: | :---: | :---: | :---: |
| Unit 7 | Systems of Equations | 15 days | 20 days |
| Unit 8 | Scatter Plots | 8 days | 15 days |
| Unit 9 | Graph Analysis of Square Root and Cube Root Functions | Time Permitting |  |
| Resources |  |  |  |
| Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017) <br> Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta <br> Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing <br> calculators and emulators |  |  |  |

## UNIT 1: ARITHMETIC VS. GEOMETRIC SEQUENCES

## Summary and Rationale

In this introductory unit, students will explore the differences and similarities between arithmetic and geometric sequences. Through these patterns, students will begin to develop an understanding of how relationships can be translated between tables, graphs, and functions. This unit forms the basis of the understanding of linear and exponential functions, which comprise two of the four primary function types explored throughout the year. It also begins the discussion of domain and range, which is a central theme of the course.

## Recommended Pacing

## 5 days

## State Standards

## Standard A-IF Interpreting Functions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the <br> integers. |

## Standard F-BF Building Functions

CPI \# $\quad$ Cumulative Progress Indicator (CPI)
2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

## Instructional Focus

## Unit Enduring Understandings

- Multiple representations enhance understanding.


## Unit Essential Questions

- Can multiple representations of the same idea exist?


## Objectives

## Students will know:

- The attributes of arithmetic and geometric sequences.


## Students will be able to:

- Recognize the patterns for arithmetic and geometric sequences and complete the associated tables
- Graph tables for arithmetic and geometric sequences and recognize patterns effect on graph
- Develop the concept of domain and range based off of table and graph.
- $\quad\left({ }^{* *}\right)$ Develop an expression to represent the $n$th term of a sequence (arithmetic, geometric, and other)


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta
Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 2: LINEAR FUNCTIONS, GRAPHS AND TABLES

## Summary and Rationale

In this unit, students will explore the critical attributes of linear functions and be able to make connections between tables, graphs, or equations for any linear function. Modeling and applications for linear functions will be explored so that students can develop a comprehensive understanding of rate of change, what it means, and how it is calculated. Domain and range will be analyzed in tables, graphs, and as restricted parameters in real-world scenarios. Students will also work with graph transformations in order to build an understanding that will be applied to other functions with more complexity later in the course. Finally, complete graph analysis of linear functions will be undertaken so that students build the habit of understanding the critical attributes of the graph.

## Recommended Pacing

## 15 days

## State Standards

## Standard F-BF Building Functions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| $\mathbf{0}$ | Build a function that models a relationship between two quantities. Build new functions from existing <br> functions. |
| $\mathbf{1}$ | Write a function that describes a relationship between two quantities. <br> a. Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| $\mathbf{3}$ | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific <br> values of $k$ (both positive and negative); find the value of k given the graphs. Experiment with cases and <br> functions from their graphs and algebraic expressions for them. |
| Standard F-IF Interpreting Functions |  |
| $\mathbf{C P I} \#$ | Cumulative Progress Indicator (CPI) |
| $\mathbf{0}$ | Understand the concept of a function and use function notation; Interpret functions that arise in <br> applications in terms of the context; Analyze functions using different representations. |
| $\mathbf{1}$ | Understand that a function from one set (called the domain) to another set (called the range) assigns to <br> each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its <br> domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of <br> the equation $y=f(x)$. |
| $\mathbf{2}$ | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use <br> function notation in terms of a context. |
| $\mathbf{4}$ | For a function that models a relationship between two quantities, interpret key features of graphs and <br> tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the <br> relationship. Key $f e a t u r e s ~ i n c l u d e: ~ i n t e r c e p t s ; ~ i n t e r v a l s ~ w h e r e ~ t h e ~ f u n c t i o n ~ i s ~ i n c r e a s i n g, ~ d e c r e a s i n g, ~$ <br> positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. $\star$ |


| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. |
| :---: | :---: |
| 6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| 7a | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> a. Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| Standard F.LE Linear, Quadratic and Exponential Models |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1a | Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |
| 1b | Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. |
| 2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| 5 | Interpret the parameters in a linear or exponential function in terms of a context. |
|  | Instructional Focus |
| Unit Enduring Understandings |  |
| - The value of a particular representation depends on its purpose. <br> - Perspective builds understanding. |  |
| Unit Essential Questions |  |
| - How is change related to behavior? <br> - Can situations be represented graphically? |  |
| Objectives |  |
| Students will know: <br> - How to identify whether a function is linear. <br> - The concepts of domain and range. <br> - The meaning of slope (rate of change) and the $y$-intercept (initial value) and their impact on the parent function. <br> - Various ways to graph a linear function. <br> - The solution to a system of linear equations is the point of their intersection. |  |

## Students will be able to:

- Determine if an arithmetic sequence is a function (Is the table one-to one? and Does the graph pass the vertical line test?)
- Connect the slope and $y$-intercept to the rate of change and initial value (identify slope and $y$-intercept from both a table and a graph; include horizontal and vertical lines in the analysis)
- Write a linear function in function form $(f(x)=m x+b)$ from table and graph
- Expand function notation to find $f(k), f(x)+k, k f(x), f(k x), f(x+k)$ using the equation and the table
- Transformations of Linear graphing [ $f(k), f(x)+k, k f(x), f(k x), f(x+k)$ ]
- Write functions in function form (given a point and the slope, given two points, real-life scenarios); produce answers in both standard form and intercept form.
- Review graphing linear functions using a table, transformations, slope and $y$-intercept, or intercepts. Intercepts can be given or can be analyzed from a table.
- Graph linear functions with a restricted domain
- Understand the meaning of domain and range and identify domain and range in linear and non-linear graphs
- Graph two linear functions and understand the meaning of their intersection
- Solve literal equations for a given variable.
- Solve word problems associated with linear functions
- Review solving equations by determining $x$-intercepts and $y$-intercept; as well as determining $f(x)=a$
- ( $\left.{ }^{* *}\right)$ Evaluate composition of functions


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 3: EXPONENTIAL FUNCTIONS, GRAPHS, AND TABLES

| Summary and Rationale |  |
| :---: | :---: |
| In this unit, students will explore the critical attributes of exponential functions and be able to make connections between tables, graphs, or equations for any exponential function. Modeling and applications for exponential functions will be explored and domain and range will be analyzed in tables, graphs, and as restricted parameters in real-world scenarios. Students will continue to work with graph transformations and will be introduced to the terms asymptote, growth factor, and decay factor. Comparisons between linear functions and exponential functions will be examined so that students remain aware of different types of rates of change. Finally, complete graph analysis of exponential functions will be undertaken so that students continue to build the habit of understanding the critical attributes of the graph. |  |
| Recommended Pacing |  |
| 10 days |  |
| State Standards |  |
| Standard F-BF Building Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 0 | Build a function that models a relationship between two quantities. Build new functions from existing functions. |
| 1 | Write a function that describes a relationship between two quantities. <br> a. Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| 3 | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. |
| Standard F-IF Interpreting Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 0 | Understand the concept of a function and use function notation; Interpret functions that arise in applications in terms of the context; Analyze functions using different representations. |
| 1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$. |
| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |


| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. $\star$ |
| :---: | :---: |
| 7 e | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <br> e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |
| 8b | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. <br> Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^{t}, y=(0.97)^{t}, y=(1.01)^{12 t}, y=(1.2)^{t / 10}$, and classify them as representing exponential growth or decay. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| Standard F.LE Linear, Quadratic and Exponential Models |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 0 | Linear, Quadratic, \& Exponential Models Construct and compare linear, quadratic, and exponential models and solve problems; Interpret expressions for functions in terms of the situation they model. |
| 1a | Distinguish between situations that can be modeled with linear functions and with exponential functions. <br> a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |
| 1c | Distinguish between situations that can be modeled with linear functions and with exponential functions. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| 2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| 3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| 5 | Interpret the parameters in a linear or exponential function in terms of a context. |
|  | Instructional Focus |
| Unit Enduring Understandings |  |
| - The value of a particular representation depends on its purpose. <br> - Characteristics depend on the situation. <br> - Perspective builds understanding. |  |
| Unit Essential Questions |  |
| - How is change related to behavior? <br> - Are characteristic unique? <br> - Can situations be represented graphically? |  |

## Objectives

## Students will know:

- Geometric sequences are exponential functions.
- How to identify whether a function is exponential.
- The concepts of domain and range as it relates to exponential functions.
- The meaning of a growth or decay factor and the $y$-intercept (initial value) and their impact on the parent function.
- Various ways to graph an exponential function.
- The solution to a system of equations is their point of intersection.


## Students will be able to:

- Determine whether a geometric sequence is a function (Is the table one-to one? and Does the graph pass the vertical line test?)
- Connect the common ratio to the growth/decay factor of an exponential function
- Make a connection between the negative exponent rule and the pattern in the table of an exponential function
- $\quad\left({ }^{* *}\right)$ Write an exponential function in function form $f(x)=a b^{x}$ from a table and a graph by recognizing the pattern (include vertical, horizontal and reflections)
- Graph exponential functions with a restricted domain
- Graph a linear and an exponential on the same graph and discuss the meaning of their intersection
- Model exponential functions in real life scenarios throughout the unit. Include problem solving for initial value and growth factor and compound interest.
- Compare and contrast the characteristics of linear and exponential functions throughout the unit.


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 4: ABSOLUTE VALUE AND QUADRATIC FUNCTIONS

## Summary and Rationale

In this unit, students continue to explore different graphs and equation types by building an understanding of absolute value functions and quadratic functions. Students will explore the parent form of each function type and discover how different parameters transform each function on a coordinate plane. They will relate the two functions to each other with regard to symmetry and rate of change in order to directly compare linear and nonlinear rates. Throughout the unit, the focus will be on exploring the relationship between tables, graphs, and equations for absolute value and quadratic functions and students will continue work on graph analysis tools regarding domain, range, $\mathrm{max} / \mathrm{min}$, axis of symmetry, and intercepts.

## Recommended Pacing

## 20 days

## State Standards

## Standard F-BF Building Functions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| $\mathbf{0}$ | Build a function that models a relationship between two quantities. Build new functions from existing <br> functions. |
| $\mathbf{1}$ | Write a function that describes a relationship between two quantities. <br> a. Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| $\mathbf{3}$ | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific <br> values of k (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and <br> illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd <br> functions from their graphs and algebraic expressions for them. |
| Standard F-IF Interpreting Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $\mathbf{0}$ | Understand the concept of a function and use function notation; Interpret functions that arise in <br> applications in terms of the context; Analyze functions using different representations. |
| $\mathbf{1}$ | Understand that a function from one set (called the domain) to another set (called the range) assigns to <br> each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its <br> domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of <br> the equation $y=f(x)$. |
| $\mathbf{2}$ | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use <br> function notation in terms of a context. |
| $\mathbf{4}$ | For a function that models a relationship between two quantities, interpret key features of graphs and <br> tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the <br> relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, <br> positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. $\star$ |


| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it <br> describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ <br> engines in a factory, then the positive integers would be an appropriate domain for the function. $\star$ |
| :--- | :--- |
| 6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over <br> a specified interval. Estimate the rate of change from a graph. $\star$ |
| 7a | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and <br> using technology for more complicated cases. $\star$ <br> a. Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| 7b | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and <br> using technology for more complicated cases. $\star$ <br> b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute <br> value functions. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, <br> numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and <br> an algebraic expression for another, say which has the larger maximum. |
| Standard F.LE Linear, Quadratic and Exponential Models |  |$|$| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 0 | Linear, Quadratic, \& Exponential Models Construct and compare linear, quadratic, and exponential models <br> and solve problems; Interpret expressions for functions in terms of the situation they model. |
| Standard A.REI Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the <br> coordinate plane, often forming a curve (which could be a line). |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ <br> intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using <br> technology to graph the functions, make tables of values, or find successive approximations. Include cases <br> where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic <br> functions. $\star$ |
| Unit Enduring Understandings |  |
| - Comparisons create characteristics. |  |
| The value of a particular representation depends on its purpose. |  |
| - Perspective builds understanding. |  |

## Objectives

## Students will know:

- The characteristics of an absolute value and quadratic function.
- How to identify whether a function is absolute value or quadratic.
- The concepts of domain and range as it relates to absolute value and quadratic functions.
- The meaning of a vertex, axis of symmetry, and the $y$-intercept (initial value) and their impact on the parent function.
- How to graph absolute value functions and quadratic functions in vertex form only.
- The solution to a system of equations is their point of intersection.


## Students will be able to:

- Solve absolute value equations and inequalities and graph on a number line.
- Determine whether an absolute value graph is a function (Is the table one-to one? and Does the graph pass the vertical line test?)
- Expand function notation to explore $f(k), f(x)+k, k f(x), f(k x), f(x+k)$ using the equation and the table
- Graph absolute value functions using the transformations $[f(x)+k, k f(x), f(k x), f(x+k)]$
- Write an absolute value function in function form $(f(x)=a|x-h|+k)$ from a table and a graph
- Identify the vertex and $a$-value of each function and solve for the $x$ - and $y$-intercepts
- Graph absolute value functions that have a restricted domain
- Graph two functions (linear, exponential, or absolute value) and discuss the meaning of their intersections
- Model absolute value functions in real life scenarios throughout the unit
- Determine whether a quadratic graph is a function (Is the table one-to one? and Does the graph pass the vertical line test?)
- Expand function notation to explore $f(k), f(x)+k, k f(x), f(k x), f(x+k)$ using the equation and the table
- Graph quadratic functions using the transformations $[f(x)+k, k f(x), f(k x), f(x+k)]$
- Write a quadratic function in function form $\left(f(x)=a(x-h)^{2}+k\right)$ from a table and a graph
- Graph quadratic functions that have a restricted domain
- Graph two functions (linear, exponential, absolute value, or quadratics) and discuss the meaning of their intersections
- Model quadratic functions in real life scenarios throughout the unit
- Calculate and interpret the average rate of change of a function over a specified interval.
- Compare and contrast linear, exponential, absolute value, and quadratic functions from tables, graphs, and equations, throughout the unit.


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 5: POLYNOMIAL AND RADICAL OPERATIONS

## Summary and Rationale

In this unit, students perform operations with polynomials and radical expressions. Including in these inquiries is the development of the properties of exponents (product, quotient, power of a power, zero) and their algebraic proof. Students will also simplify and explore radicals, understanding how the index and radicand are used in simplifying expressions. Operations with polynomials will prime students for later work with factoring and analysis of quadratic functions written in standard form. Applications with polynomial expressions and radicals will highlight how these quantities are used in real-life models.

## Recommended Pacing

## 15 days

## State Standards

| Standard A.APR Arithmetic with Polynomial and Rational Expressions |  |
| :---: | :---: |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| Standard A.SSE Seeing Structure in Expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1a | Interpret expressions that represent a quantity in terms of its context. <br> a. Interpret parts of an expression, such as terms, factors, and coefficients. |
| 2 | Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$. |
| 3 c | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. <br> Use the properties of exponents to transform expressions for exponential functions. For example the expression $1.15^{t}$ can be rewritten as $(1.151 / 12) 12 t \approx 1.01212 t$ to reveal the approximate equivalent monthly interest rate if the annual rate is $15 \%$. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
|  | mparisons create characteristics. <br> value of a particular representation depends on its purpose. spective builds understanding. |
| Unit Essential Questions |  |
|  | comparisons create understanding? <br> $w$ is change related to behavior? <br> n situations be represented graphically? |

## Objectives

## Students will know:

- The properties of exponents (product, quotient, power of a power, zero).
- The meaning of like terms as related to expressions with exponents and expressions using radicals.
- The definition of a polynomial and how to determine the degree of a polynomial expression.
- How to simplify radical expressions.
- The Pythagorean Theorem and how it relates to radical expressions and polynomials.


## Students will be able to:

- Name and identify polynomial expressions by their degree
- Perform operations with polynomials (add, subtract and multiply). Multiplication is limited to: monomial*polynomial and binomial*binomial. The properties of exponents review is embedded within this content, but does not include negative exponents.
- Simplify radicals
- (**) Simplify radicals with variables (write solutions with absolute value when applicable)
- Perform operations with radicals (add, subtract, multiply, and divide)
- Rationalize denominators
- Apply operations of radicals and polynomials to real-life scenarios (include Pythagorean Theorem, area and perimeter)


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 6: RELATING THE ROOTS OF A QUADRATIC FUNCTION TO ITS GRAPH

## Summary and Rationale

In this unit, students build on their understanding of quadratic functions and their graphs to formalize the relationship between solutions to quadratic equations and the $x$-intercepts (roots) of graphs. Students will use a variety of solving methods to explore quadratic equations including factoring, taking square roots, completing the square, and the quadratic formula. With these methods, students will be able to relate the three forms of a quadratic equation to each other and will be able to build quadratic equations when given the roots. Comparisons between quadratic and linear functions will continue to be made as students explore graphs with one $x$-intercept, two x-intercepts, and zero $x$-intercepts.

## Recommended Pacing

## 25 days

## State Standards

| Standard F.BF Building Functions |  |
| :--- | :--- |
| $\mathbf{C P I} \#$ | Cumulative Progress Indicator (CPI) |
| $\mathbf{1}$ | Write a function that describes a relationship between two quantities. $\star$ |
| $\mathbf{3}$ | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific <br> values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and <br> illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd <br> functions from their graphs and algebraic expressions for them. |
| Standard F.IF Interpreting Functions |  |
| $\mathbf{C P I} \#$ | Cumulative Progress Indicator (CPI) |
| $\mathbf{1}$ | Understand that a function from one set (called the domain) to another set (called the range) assigns to <br> each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its <br> domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of <br> the equation $y=f(x)$. |
| $\mathbf{2}$ | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use <br> function notation in terms of a context. |
| $\mathbf{4}$ | For a function that models a relationship between two quantities, interpret key features of graphs and <br> tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the <br> relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, <br> positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. $\star$ |
| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it <br> describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ <br> engines in a factory, then the positive integers would be an appropriate domain for the function. $\star$ |
| $7 a$ | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and <br> using technology for more complicated cases. $\star$ <br> a. Graph linear and quadratic functions and show intercepts, maxima, and minima. |


| 8a | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. <br> a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |
| :---: | :---: |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| Standard A.APR Arithmetic with Polynomial and Rational Expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. |
| 4 | Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $\left(x^{2}+y^{2}\right)^{2}=\left(x^{2}-y^{2}\right)^{2}+(2 x y)^{2}$ can be used to generate Pythagorean triples. |
| Standard A.REI Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 4a | Solve quadratic equations in one variable. <br> a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form. |
| 4b | Solve quadratic equations in one variable. <br> b. Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b$. |
| 10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
| - Rules foster comprehension. <br> - Different perspectives lead to understanding. |  |
| Unit Essential Questions |  |
|  | y are rules necessary? <br> viewpoints change meaning? |

## Objectives

## Students will know:

- The characteristics of a quadratic function.
- The three forms of a quadratic function and how they relate to each other and to their graphs.
- The various methods for solving a quadratic function and how to choose an appropriate method to solve a given problem.
- That connections between roots, intercepts, and solutions, and how they relate the graph and the function.


## Students will be able to:

- Factor using the greatest common factor, 4+ terms by grouping, trinomials by grouping where $a=1$ and $a \neq$ 1 (including perfect square trinomials), and difference of squares
- (**) Factor using $u$-substitution
- Find the roots/zeroes of quadratics by solving using factoring, square roots, quadratic formula, completing the square (with $a=1$ or GCF that creates $a=1$ with focus on perfect square trinomial)
- Write the equation of a quadratic function given its roots and based on a given range for the function.
- Determine the number of solutions for an equation and connect that information to the equation
- Connect the graph of a quadratic function to its roots
- Graph quadratic functions in vertex form, standard form, and intercept form. Standard form equations should be factorable and the vertex should be discovered by using $x$-intercepts
- $\quad\left(^{* *}\right)$ Graph quadratic functions in standard form by using $\left.x=-b / 2 a\right)$
- Convert between the three forms of a quadratic function
- Model quadratic functions with real life scenarios throughout the unit (ex. time vs. height, area)
- $\quad\left({ }^{* *}\right)$ Understand that when solving a quadratic equation, the radicand must be greater than or equal to zero.


## Resources

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## UNIT 7: SYSTEMS OF EQUATIONS

## Summary and Rationale

In this unit, students will explore the relationships between function types by graphing and solving systems of equations. Students will understand that regardless of the number of functions in a system or the function types used, solutions are defined as the intersection(s) of the given functions on the graph. Students will learn formal algebraic methods to solve systems and determine whether given values represent solutions to a system. Systems of inequalities will also be explored during the unit and students will be able to recognize scenarios in which a system has infinitely many solutions as opposed to a finite number of intersections. Modeling will be used throughout the unit to highlight the many scenarios that can be represented by systems of equations.

## Recommended Pacing

15 Days

## State Standards

## Standard A.REI Reasoning with Equations and Inequalities

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that <br> equation and a multiple of the other produces a system with the same solutions. |
| 6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear <br> equations in two variables. |
| 10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the <br> coordinate plane, often forming a curve (which could be a line). |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ <br> intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using <br> technology to graph the functions, make tables of values, or find successive approximations. Include cases <br> where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic <br> functions. $\star$ |
| 12 | Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the <br> case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as <br> the intersection of the corresponding half-planes. |

Instructional Focus

## Unit Enduring Understandings

- Solutions depend on parameters.
- There may not always be one solution to a problem.


## Unit Essential Questions

- How do conditions impact solutions?
- When can multiple solutions exist?


## Objectives

## Students will know:

- Various ways to solve a system algebraically and graphically.
- That the solution to a system of equations is the intersection point(s) of the functions.
- Systems of linear equations can have zero, one, or infinitely many solutions.
- Systems of equations that include exponential, absolute value, or quadratic functions may have zero, one, two, three, four, or infinitely many solutions.
- Two variable inequalities have infinitely many solutions.


## Students will be able to:

- Identify whether a coordinate is a solution to the system of equation graphically and from a set of equations.
- Graph two linear equations and determine whether they have one solution, no solution, or infinitely many solutions and relate that information to the equation.
- Solve a system of two linear equations using substitution and elimination (linear combination).
- Identify the number and types of solutions for all pairs of function types.
- Create a system of equations that has a given number of solutions or specific solution coordinate.
- Apply solving systems of equations to real life applications.
- $\quad\left(^{* *}\right)$ Solve systems of equations in three variables.
- Discuss the meaning of an inequality.
- Substitute an ordered pair into a two variable inequality to determine whether it is a solution.
- Graph a single linear inequality and discuss the solutions.
- Graph a system of linear inequalities and shade the solution to the system.


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017) Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 8: SCATTER PLOTS

## Summary and Rationale


#### Abstract

In this unit, students will explore scatter plots and trend lines. Through this inquiry, students will collect, plot, and analyze data by hand and through the use of technology in order to see patterns of data. They will also use trend lines to make predictions about certain outcomes and discuss how data sets can be interpreted in a number of ways. The correlation coefficient will be discussed and calculated so that students can compare data that is highly correlated to data that is not, and also relate their inquiry to functions previously discussed during the course. This unit is very focused on models and real-life applications, so students will use relevant data sets to undertake their work.


## Recommended Pacing

## 8 Days

## State Standards

## Standard S.ID Interpreting Categorical and Quantitative Data

CPI \# $\quad$ Cumulative Progress Indicator (CPI)

| 6 a | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. <br> a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use <br> given functions or choose a function suggested by the context. Emphasize linear, quadratic, and <br> exponential models. |
| :--- | :--- |
| 6 b | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. <br> b. Informally assess the fit of a function by plotting and analyzing residuals. |
| 6 c | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. <br> c. Fit a linear function for a scatter plot that suggests a linear association. |

## Instructional Focus

## Unit Enduring Understandings

- Analyzing trends predict behavior.


## Unit Essential Questions

- Can understanding a relationship help make a prediction?
- Why are relationships necessary?


## Objectives

## Students will know:

- That relationships can be modeled and analyzed using scatter plots.
- The relationship of a scatter plot can be represented with a function


## Students will be able to:

- Analyze linear scatter plots, discuss correlation, find the line of best fit, and use the line of best fit to make predictions
- Analyze scatter plots and determine the best non-linear function type to represent the data, use technology to find the most appropriate equation
- Find the regression function and correlation coefficient by using technology.
- Determine the equation given that best represents the data given a scatter plot and multiple equations,
- Use functions to make predictions from a scatter plot.


## Resources

Core Text: Piscataway Township Schools Algebra 1 Course Materials (developed 2017)
Suggested Resources: Algebra 1 (2014). Kanold, Burger, et al. Houghton Mifflin Harcourt; desmos.com; Kuta Infinite Algebra 1; deltamath.com; https://parcc.pearson.com/practice-tests/math; collegeboard.org; graphing calculators and emulators

## UNIT 9: GRAPH ANALYSIS OF SQUARE ROOT AND CUBE ROOT FUNCTIONS

| Summary and Rationale |  |
| :---: | :---: |
| In this unit, students will use the graph analysis techniques learned in earlier units to explore square root and cube root functions. Students will use parent function transformations to examine how these new function types behave on a coordinate plane and will examine tables, equations, and graphs to discuss domain, range, endpoints, and other pertinent information. Throughout this unit, students will gain an understanding of inverse functions and the relationships that exist between those function types. Models will be used to highlight real-life applications of each type of function. |  |
| Recommended Pacing |  |
| Time permitting |  |
| State Standards |  |
| Standard F.BF Building Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 3 | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. |
| 4c | Find inverse functions. <br> (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. |
| Standard F.IF Interpreting Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$. |
| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. |
| 7b | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |


|  | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute <br> value functions. |
| :--- | :--- |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, <br> numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and <br> an algebraic expression for another, say which has the larger maximum. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
| - $\quad$ Patterns create clarity and understanding. |  |
| Unit Essential Questions |  |
| - $\quad$ Can change create patterns? |  |
| Objectives |  |
| Students will know: <br> - The characteristics of a square root and cube root functions. <br> - The connection between the index of a radical and a rational exponent. <br> - $\quad$ How to convert an expression between radical form and exponential form. <br> Students will be able to: <br> - Understand that square roots are expressions to the $1 / 2$ power and cube roots are expressions to the $1 / 3$ <br> power. <br> - Transform square root functions $[f(x)+k, k f(x), f(k x), f(x+k)]$. <br> - Write a square root function in function form ( $f(x)=a \sqrt{x-h}+k)$ from a table and a graph. <br> - $\quad$ Explore inverses functions and their effect on the graph. |  |
| - Convert an expression between radical form and exponential form. |  |
| - Compare and contrast the characteristics of square root and cube root functions and graphs to linear, |  |

